2014 One-Day Workshop on Numerical Partial Differential Equations

(Activity of Numerical PDE Group, TWSIAM)

時間：November 13, 2014
地點：國立高雄師範大學數學系 燕巢校區致理大樓 MA803
主辦單位：國立高雄師範大學數學系
贊助單位：科技部自然司數學研究推動中心

偏微分方程是建構科學與工程數學模型常用的工具，一般情況下這類數學模型僅能藉由數值方法求其近似解，因此發展穩定有效的數值方法求解偏微分方程在理論及應用上，一直是個重要的研究議題。本研討會主要邀請這個領域幾位具代表性的學者介紹其目前最新的研究進展，歡迎相關人員和學生共同參與。講員有：

楊肅煜　教授　　國立中央大學數學系
Title: An efficient stabilized linear finite element method for solving reaction-convection-diffusion equations

施因澤　教授　　國立中興大學應用數學系
Title: Tailored finite point method for numerical solutions of singular perturbed eigenvalue problems

舒宇宸　教授　　國立成功大學數學系
Title: Accurate gradient approximation for complex interface problems in 3D by an improved coupling interface method

曾昱豪　教授　　國立高雄大學應用數學系
Title: An immersed boundary method for endocytosis

黃杰森　教授　　國立中山大學應用數學系
Title: Computing eigenmodes of elliptic operators using increasingly flat radial basis functions

Organizers: 李俊憲・國立高雄師範大學數學系
吳恭儉・國立高雄師範大學數學系

TEL : (07) 7172930 轉 6813 (李), 6827 (吳)
E-Mail: chli@nknucc.nknu.edu.tw ; kcwu@nknucc.nknu.edu.tw

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主辦單位

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贊助單位

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Organized by

Chun-Hsien Li (李俊憲)

Kung-Chien Wu (吳恭儉)

Schedule of Programs

Place：高雄師大燕巢校區致理大樓 MA803 e 化教室

Nov. 13	Speaker	Title
10:30 – 11:20	**Suh-Yuh Yang (楊肅煜)**	**An efficient stabilized linear finite element method for solving reaction-convection-diffusion equations**
11:20 – 11:30	**Break**	
11:30 – 12:20	**Yu-Hau Tseng (曾昱豪)**	**An immersed boundary method for endocytosis**
12:20 – 14:00	**Lunch**	
14:00 – 14:50	**Chieh-Sen Huang (黃杰森)**	**Computing eigenmodes of elliptic operators using increasingly flat radial basis functions**
14:50 – 15:00	**Break**	
15:00 – 15:50	**Yintzer Shih (施因澤)**	**Tailored finite point method for numerical solutions of singular perturbed eigenvalue problems**
15:50 – 16:00	**Break**	
16:00 – 16:50	**Yu-Chen Shu (舒宇宸)**	**Accurate Gradient Approximation for Complex Interface Problems in 3D by an Improved Coupling Interface Method**

An efficient stabilized linear finite element method for solving reaction-convection-diffusion equations

Suh-Yuh Yang

Department of Mathematics, National Central University
Taoyuan 32001, Taiwan
E-mail: syyang@math.ncu.edu.tw

Abstract

In this talk, we will propose an efficient stabilized linear finite element method (FEM) for solving reaction-convection-diffusion equations with arbitrary magnitudes of reaction and diffusion. The key feature of the new method is that the test function in the stabilization term is taken in the adjoint-operator-like form $-\varepsilon\Delta v - (\boldsymbol{a}\cdot\nabla v)/\gamma + \sigma v$, where the parameter γ is appropriately designed to adjust the convection strength to achieve high accuracy and stability. We derive the stability estimates for the finite element solutions and establish the explicit dependence of L^2 and H^1 error bounds on the diffusivity, module of the convection field, reaction coefficient and the mesh size. The analysis shows that the proposed method is suitable for a wide range of mesh Péclet numbers and mesh Damköhler numbers. More specifically, if the diffusivity ε is sufficiently small with $\varepsilon < \|\boldsymbol{a}\|h$ and the reaction coefficient σ is large enough such that $\|\boldsymbol{a}\| < \sigma h$, then the method exhibits optimal convergence rates in both L^2 and H^1 norms. However, for a small reaction coefficient satisfying $\|\boldsymbol{a}\| \geq \sigma h$, the method behaves like the well-known streamline upwind/Petrov-Galerkin formulation of Brooks and Hughes. Several numerical examples exhibiting boundary or interior layers are given to demonstrate the high performance of the proposed method. Moreover, we apply the developed method to time-dependent reaction-convection-diffusion problems and simulation results show the efficiency of the approach. This is joint work with Po-Wen Hsieh.

An immersed boundary method for endocytosis

Yu-Hau Tseng

Department of Applied Mathematics, National University of Kaohsiung
Kaohsiung 81148, Taiwan
E-mail: yhtseng@nuk.edu.tw

Huaxiong Huang

Department of Mathematics and Statistics, York University
Toronto, Ontario M3J 1P3, Canada

Centre for Mathematical Medicine, Fields Institute
Toronto, Ontario M5T 3J1, Canada

Abstract

Endocytosis is one of the cellular functions for capturing (engulfing) vesicles or microorganisms. Understanding the biophysical mechanisms of this cellular process is essential from a bioengineering point of view since it will provide guidance for developing effective targeted drug delivery therapies. In this talk, we propose an immersed boundary (IB) method that can be used to simulate the dynamical process of this important biological function. In our model, membranes of the vesicle and the cell are treated as Canham-Helfrich Hamiltonian interfaces. The membrane-bound molecules are modelled as insoluble surfactants such that the molecules after binding are regarded as a product of a chemical reaction. Our numerical examples show that the immersed boundary method is a useful simulation tool for studying endocytosis, where the roles of interfacial energy, fluid flow and viscous dissipation in the success of the endocytosis process can be investigated in detail. A distinct feature of our IB method is the treatment of the two binding membranes that is different from the merging of fluid-fluid interfaces. Another important feature of our method is the strict conservation of membrane-borne receptors and ligands, which is important for predicting the dynamics of the endocytosis process.

Computing eigenmodes of elliptic operators using increasingly flat radial basis functions

Chieh-Sen Huang

Department of Applied Mathematics, National Sun Yat-sen University
Kaohsiung 80424, Taiwan
E-mail: huangcs@math.nsysu.edu.tw

Abstract

Solving eigenmodes problem for elliptic operator using radial basis functions (RBFs) was proposed by Platte and Driscoll. They convert the eigenmodes problem to an eigenpairs problem of a finite dimensional matrix. In the 1-D case, we formulate a new approach by using Lagrange interpolating polynomials as eigenfunctions. We also prove with simple conditions on the basis function, two approaches converge when increasingly flat BRFs are being used. These results are supported by numerical examples.

Tailored finite point method for numerical solutions of singular perturbed eigenvalue problems

Yintzer Shih

Department of Applied Mathematics, National Chung Hsing University
Taichung 40227, Taiwan
E-mail: yintzer_shih@email.nchu.edu.tw

Abstract

We propose two variants of tailored finite point (TFP) methods for discretizing two dimensional singular perturbed eigenvalue (SPE) problems. A continuation method and an iterative method are exploited for solving discretized systems of equations to obtain the eigen-pairs of the SPE. We study the analytical solutions of two special cases of the SPE, and provide an asymptotic analysis for the solutions. The theoretical results are verified in the numerical experiments. The numerical results demonstrate that the proposed schemes effectively resolve the delta function like of the eigenfunctions on relatively coarse grid.

Accurate gradient approximation for complex interface problems in 3D by an improved coupling interface method

Yu-Chen Shu

Department of Mathematics, National Cheng Kung University
Tainan 70101, Taiwan
E-mail: ycshu@mail.ncku.edu.tw

Abstract

Most elliptic interface solvers become complicated for complex interface problems at those "exceptional points" where there are not enough number of neighboring interior points for high order interpolation. Such complication increases especially in three dimensions. Usually, the solvers are thus reduced to low order accuracy. In this talk, we classify these exceptional points and propose two recipes to maintain order of accuracy there, aiming at improving our previous method, the coupling interface method. Yet the idea is also applicable to other interface solvers. The main idea is to have at least first order approximation for second order derivatives at those exceptional points. Recipe 1 is to use the finite difference approximation for the second order derivatives at a nearby interior grid point, whenever this is possible. Recipe 2 is to flip domain signatures and introduce a ghost state so that a second-order method can be applied. This ghost state is a smooth extension of the solution at the exceptional point from the other side of the interface. The original state is recovered by a post-processing using nearby states and jump conditions. The choice of recipes is determined by a classification scheme of the exceptional points. The method renders the solution and its gradient uniformly second-order accurate in the entire computed domain. Numerical examples are provided to illustrate the second order accuracy of the presently proposed method in approximating the gradients of the original states for some complex interfaces which we had tested previous in two and three dimensions, and a real molecule (1D63) which is double-helix shape and composed hundreds of atoms.