2010 圖論與組合學研討會

國立高雄師範大學數學系

日期：2010年5月7日

Organized by

Hsin-Hao Lai (賴欣豪)
## Schedule of Programs

**Place:** 高雄師範大學燕巢校區致理大樓 MA803

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Cyclic Sieving of Graphs

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We introduce cyclic sieving on graphs and prove four instances on classical graph classes. Our methods involve representation theory and symmetric functions.
On the $S_k$-decompositions of $\lambda K_{m,n}$ and $\lambda K_n$

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Suppose that $G$ is a graph and $H$ is a subgraph of $G$. For a positive integer $\lambda$, the $\lambda$-fold graph of $G$, denoted by $\lambda G$, is a multigraph obtained from $G$ by replacing each edge $e$ of $G$ by $\lambda$ edges with the same ends as $e$. The graph $G$ is said to be $H$-decomposable, denoted by $H|G$, if the edge set $E(G)$ of $G$ can be partitioned into subgraphs such that each subgraph is isomorphic to $H$. Such a decomposition is called an $H$-decomposition of $G$.

In this talk, we investigate the $S_k$-decompositions of $\lambda K_{m,n}$ and $\lambda K_n$. The sufficient and necessary conditions for $S_k|G$ are given, where $G = \lambda K_{m,n}$ or $G = \lambda K_n$ with $2k \leq n$. Some other results are also obtained.

**Theorem 1.** Suppose that $m$, $n$, $k$ and $\lambda$ are positive integers with $m \geq n$. Then $S_k|\lambda K_{m,n}$ if and only if one of the followings hold:

(a) $k|\lambda m$ when $n < k \leq m$.

(b) $k|\lambda mn$ when $k \leq n \leq m$.

**Theorem 2.** Suppose $2k \leq n$. Then $S_k|\lambda K_n$ if and only if $k|\lambda \left(\frac{n}{2}\right)$.

**Theorem 3.** Suppose $\frac{n}{2} < k \leq n - 1$ and $\lambda$ is even. Then $S_k|\lambda K_n$ if and only if $k|\lambda \left(\frac{n}{2}\right)$.

**Problem.** Suppose that $n$, $k$ and $\lambda$ are positive integers with $k|\lambda \left(\frac{n}{2}\right)$. Then $\lambda K_n$ is not $S_k$-decomposable if and only if $\lambda$ is odd and $\frac{n+1}{2}n - 1 > \frac{\lambda \left(\frac{n}{2}\right)}{k}$.

**References**


Equitable Coloring of $k$-regular 3-chromatic Graphs

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Consider a graph $G$. A proper $k$-coloring of $G$ is a labeling $f : V(G) \to \{1, 2, \ldots, k\}$ such that adjacent vertices have different labels. The labels are colors; the vertices of one color form a color class. The chromatic number of $G$, written $\chi(G)$, is the least $k$ such that $G$ has a proper $k$-coloring. And $G$ is 3-chromatic if $\chi(G) = 3$. Moreover, a 3-chromatic graph is $k$-regular if every vertex has degree $k$. Let $\Delta(G)$ denote the maximum degree of $G$. We say that $G$ is equitably $\Delta(G)$-colorable if there exists a proper $\Delta(G)$-coloring of $G$ such that the sizes of any two color classes differ by at most one. Clearly, $G$ can be equitably $\Delta(G)$-colorable only if $\Delta(G) \geq \chi(G)$. In this paper, we investigate the equitable $\Delta(G)$-coloring for a $k$-regular 3-chromatic graph $G$ with $k > 3$.

Keywords: Chromatic number; Regular; Equitable coloring; Maximum degree
Equitable Colorings of Kneser Graphs

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An $m$-coloring of a graph $G$ is a mapping $f : V(G) \to \{1, 2, \ldots, m\}$ such that $f(x) \neq f(y)$ for any two adjacent vertices $x$ and $y$ in $G$. The chromatic number $\chi(G)$ of $G$ is the minimum number $m$ such that $G$ is $m$-colorable. An equitable $m$-coloring of a graph $G$ is an $m$-coloring $f$ such that any two color classes differ in size by at most one. The equitable chromatic number $\chi_e(G)$ of $G$ is the minimum number $m$ such that $G$ is equitably $m$-colorable. The equitable chromatic threshold $\chi^*_e(G)$ of $G$ is the minimum number $m$ such that $G$ is equitably $r$-colorable for all $r \geq m$. It is clear that $\chi(G) \leq \chi_e(G) \leq \chi^*_e(G)$.

For $n \geq 2k + 1$, the Kneser graph $KG(n, k)$ has the vertex set consisting of all $k$-subsets of an $n$-set. Two distinct vertices are adjacent in $KG(n, k)$ if they have empty intersection as subsets. The Kneser graph $KG(2k+1, k)$ is called the Odd graph, denoted by $O_k$. In [1], the authors proved that $\chi_e(KG(n,k)) \leq \chi^*_e(KG(n,k)) \leq n - k + 1$ and $\chi_e(O_k) = \chi^*_e(O_k) = 3$. In this talk, we obtain new upper bounds for $\chi_e(KG(n,k))$. Some other results are also obtained.

**Theorem 1**[1]. Suppose that $m \geq n - k + 1$. Then $KG(n,k)$ is equitably $m$-colorable, that is, $\chi_e(KG(n,k)) \leq \chi^*_e(KG(n,k)) \leq n - k + 1$.

**Theorem 2.** Suppose $2a > n - m + 1$. If $\sum_{i=0}^{k-a} \binom{m-1}{i} \binom{n-m+1}{k-i} > \left\lceil \frac{n}{k} \right\rceil$, then $KG(n,k)$ is equitably $m$-colorable.

**Theorem 3.** Suppose $2a = n - m + 1$. If

$$\frac{1}{2} \frac{(m-1)(n-m+1)}{k-a} + \sum_{i=0}^{k-a-1} \binom{m-1}{i} \binom{n-m+1}{k-i} > \left\lceil \frac{n}{k} \right\rceil,$$

then $KG(n,k)$ is equitably $m$-colorable.

**Theorem 4**[1]. For $n \geq 5$,

$$\chi_e(KG(n,2)) = \chi^*_e(KG(n,2)) = \begin{cases} n-1 & \text{if } n \geq 7, \\ n-2 & \text{if } n = 5 \text{ or } 6. \end{cases}$$
Theorem 5[1]. For $n \geq 7$,

$$\chi_\pi(KG(n,3)) = \chi_\pi^*(KG(n,3)) = \begin{cases} 
  n - 2 & \text{if } n \geq 16, \\
  n - 3 & \text{if } 14 \leq n \leq 15, \\
  n - 4 & \text{if } 7 \leq n \leq 13.
\end{cases}$$

Theorem 6[1]. $\chi(O_k) = \chi_\pi(O_k) = \chi_\pi^*(O_k) = 3$ for $k \geq 1$.

Conjecture[1]. $\chi_\pi(KG(n,k)) = \chi_\pi^*(KG(n,k))$ for $k \geq 2$.

References


A Combinatorial Proof of the Cyclic Sieving Phenomenon for Faces of Coxeterhedra

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For a Coxeter system \((W, S)\), the subgroups \(W_J\) generated by subsets \(J \subseteq S\) are called \textit{parabolic subgroups} of \(W\). The \textit{Coxeterhedron} \(PW\) associated to \((W, S)\) is the finite poset of all cosets \(\{wW_J\}_{w \in W, J \subseteq S}\) of all parabolic subgroups of \(W\), ordered by inclusion. This poset can be realized by the face lattice of a simple polytope, constructed as the convex hull of the orbit of a generic point in \(\mathbb{R}^n\) under an action of the reflection group \(W\). For the groups \(W = A_{n-1}, B_n,\) and \(D_n\) in a case-by-case manner, we present an elementary proof of the cyclic sieving phenomenon (CSP) for faces of various dimensions of \(PW\) under the action of a cyclic group generated by a Coxeter element. This result provides a geometric, enumerative and combinatorial approach to the classical type of a theorem in [Reiner-Stanton-White, The cyclic sieving phenomenon, J. Combinatorial Theory Ser. A 108 (2004) 17–50], which is proved by an algebraic method that involves representation theory and Springer’s theorem on regular elements. In this talk, we shall give a brief introduction on the notion CSP and present the combinatorial and algebraic aspects of the CSP for faces of Coxeterhedra. This talk is based on joint work with S.-P. Eu and Y.-J. Pan.